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Electromagnetically induced transparency in asymmetric double quantum wells

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Abstract. We show how to compute nonlinear optical absorption spectra of an Asymmetric Double Quantum Well (ADQW) in the region of intersubband electronic transitions. The method uses the microscopic calculation of the dephasing due to electron-electron and electron-phonon scattering rates and the macroscopic real density matrix approach to compute the electromagnetic fields and susceptibilities. The polarization dephasing and the corrections to the Rabi frequencies due to the electron-electron interaction are also taken into account. For a proper choice of the QW widths and of the driving fields we obtain electromagnetically induced transparency. This transparency has a very narrow linewidth when a single driving field is applied resonant to the transition between the second and the third subband. In the case of two resonant driving fields or of a driving field resonant between the first and third subband we obtain a large transparency enhancement over the entire absorption spectrum. Results are given for GaAs/GaAlAs QWs and experiments are proposed.

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1 Introduction

Intersubband transitions in n-doped quantum well structures have attracted a great deal of attention in recent years, both from the theoretical [1] and the experimental point of view [2]. Nonlinear effects involving three subbands have also been experimentally studied in symmetric quantum wells [3]. Sadeghi *et al.* [4] have shown that in the case of an asymmetric quantum well an electromagnetically induced transparency (EIT) may be obtained with appropriate driving fields, provided that the coherence between the subbands considered is preserved for a sufficiently long time [5–9].

The phenomenon of EIT has been deeply studied in atomic physics [10], starting from its observation in sodium vapours [11], where the hyperfine split s-states transitions to $3p$ states are shown to interfere destructively at resonance. It has been later found also in strontium vapours, where the driving field coupling a metastable empty state to the final state produces transparency in the probe beam [12]. Many consequences of EIT have been studied and experimentally verified [10]; particularly significant are the enormous reductions of the group velocity [13] and the expected novelties in the light propagation in a medium [14] and in the Cerenkov light emission [15].

In what follows we study the nature of the EIT phenomenon in the case of intersubband transitions in an n-type Asymmetric Double Quantum Well (ADQW). Nonlinear optical processes are considered, taking into account the coherent amplitude interference of the transition probabilities in the electromagnetic fields. We show that the EIT effect takes place for sufficiently intense driving fields under appropriate conditions, which depend on the electron-electron and electron-phonon scattering. Our approach is similar to that of of Sadeghi *et al.* [4], but considers explicitly the k-dependence of all the scattering processes, including the polarization dephasing and the corrections to the Rabi frequencies described by Lindberg and Koch [16]. We will distinguish two cases, one with a control driving field and a probe field, and a second case with two coherent driving fields which produce population trapping, and a weak probe field. The second case is analogous to the one studied by Narducci *et al.* [17] in three level atomic systems.

We make use of the density matrix approach [18–21], and consider the dephasing rates due to electron-electron and electron-phonon interaction. This enables us to study

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the effects of the coherence between the external electromagnetic fields and the motion of electrons in the conduction subbands, and to clarify the role of specific scattering processes in the possibility to observe the EIT. With detailed calculations on a specific example we prove that EIT can be obtained with one driving field and a broad transparency enhancement obtains with two driving fields.

The paper is organized as follows. In Section 2 we adopt the Stahl density matrix approach to obtain the constitutive equations for the case of three QW subbands. The constitutive equations contain terms which describe the scattering processes. In Section 3 we show how to evaluate all the relevant contributions to the dephasing rates due to the electron-electron and electron-phonon scattering, and thus obtain a closed system of equations for the time-dependent density matrix elements. In Section 4 we present the constitutive equations in Fourier space and describe their properties. In Section 5 we give the absorption spectrum and the results for a specific case of ADQWs in the GaAs/GaAlAs system, such that electrons in the excited subbands have appropriate lifetimes to observe EIT. In Section 6 we present our conclusions and discuss experimental consequences.

2 Constitutive equations

We consider the conduction band of an asymmetric double quantum well with three subbands corresponding to the energies $\hbar\omega_1, \hbar\omega_2$ and $\hbar\omega_3$. They are represented in real space as an ensemble of three levels localized at the atomic sites of the two-dimensional Quantum Well, in accordance with Stahl's model [18]. The density matrix of the system can be described in terms of the operators

$$\hat{C}_{ij}^{nm} = \hat{c}_i^{n\dagger}\hat{c}_j^m, \tag{1}$$

where $\hat{c}_i^{n\dagger}$ denotes the creation operator for electrons at the site i in the subband n, and $\hat{c}_j^m$ is the annihilation operator for the electrons at the site j in the subband m. We start by considering a simple Hamiltonian in which the electrons occupy their states as independent particles

$$H = H_{\rm TL} + H_{\rm M} + H_{\rm EM}, \tag{2}$$

where

$$H_{TL} = \hbar\omega_1 \sum_i \hat{C}_{ii}^{11} + \hbar\omega_2 \sum_i \hat{C}_{ii}^{22} + \hbar\omega_3 \sum_i \hat{C}_{ii}^{33}, \tag{3}$$

is the Hamiltonian of the three levels system at a given position, $H_{\rm M}$ describes the possibility of motion of electrons from one site to another, which in the effective mass approximation corresponds to the kinetic energy terms, $H_{\rm EM}$ denotes the interaction with the external electromagnetic field, which in the Λ configuration indicated in Figure 1 is

$$\begin{aligned} H_{\rm EM} = &-\sum_{ij} \mathbf{E}_{ij}^1 \left(\mathbf{M}_{ij}^{31}\hat{C}_{ij}^{31} + \mathbf{M}_{ji}^{13}\hat{C}_{ji}^{13}\right) \\ &-\sum_{ij} \mathbf{E}_{ij}^2 \left(\mathbf{M}_{ij}^{32}\hat{C}_{ij}^{32} + \mathbf{M}_{ji}^{23}\hat{C}_{ji}^{23}\right), \end{aligned} \tag{4}$$

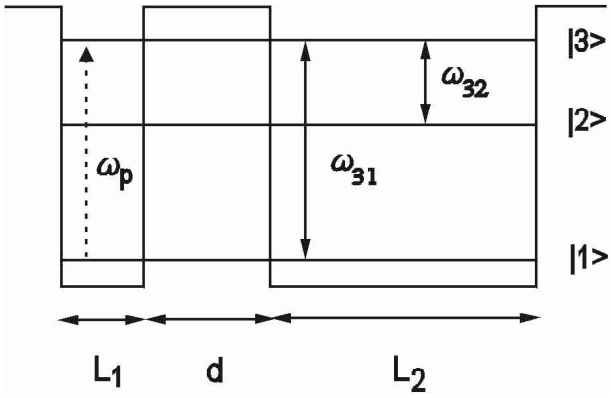

Fig. 1. Schematic diagram of the energy potentials and of the intersubband transitions in an Asymmetric Double Quantum Well. The continuous lines indicate intense control beams and the dotted line indicates the probe beam.

where $\mathbf{M}_{ij}^{nm}$ represents the dipole element for the transition between electrons located at sites i and j in the subbands n and m, $\mathbf{E}_{ij}^n$ are the electric fields of the electromagnetic waves resonant with the transitions $n \to 3$ taken at a mean point between sites i and j

$$\begin{aligned} E = E_{ij}^1 + E_{ij}^2 = &\tfrac{1}{2}\tilde{E}^1 \left(\tfrac{\boldsymbol{\rho}_i + \boldsymbol{\rho}_j}{2}\right) \mathrm{e}^{-\mathrm{i}\Omega_1 t} \\ &+\tfrac{1}{2}\tilde{E}^2 \left(\tfrac{\boldsymbol{\rho}_i + \boldsymbol{\rho}_j}{2}\right) \mathrm{e}^{-\mathrm{i}\Omega_2 t} + \text{ c.c.} \end{aligned} \tag{5}$$

We observe that in the Hamiltonian (2) the Coulomb interaction among the electrons is neglected. In the Hartree-Fock approximation this interaction would introduce a correction in the energy levels and in the intensity of the interaction with the electromagnetic field, as shown by Haug and Koch [22]. The intrasubband contributions have been shown explicitly by Huang *et al.* [23] to influence the position and dispersion of the subbands. Since our problem is not to obtain the best calculation of the electronic states, and the effect of the non parabolicity of the subbands is irrelevant at low electron densities, we feel justified in neglecting the above intrasubband corrections. We will instead consider in Section 4 the correction to the transition probabilities which produces generalized Rabi frequencies. The irreversible dephasing contributions produced by the Coulomb interaction beyond the Hartree-Fock approximation will be explicitly described and included in Sections 3 and 4, where also the effects of the electron-phonon interaction will be considered.

The equations of motion for the operators $\hat{C}_{ij}^{nm}$ can be obtained from the Heisenberg equation with an incoherent relaxation contribution as

$$\frac{\partial \hat{C}_{ij}^{nm}}{\partial t} = \frac{\mathrm{i}}{\hbar}[H, \hat{C}_{ij}^{nm}] + \left.\frac{\partial \hat{C}_{ij}^{nm}}{\partial t}\right|_{\rm incoh}. \tag{6}$$

The last term of equation (6) includes all those contributions due to the radiation field, the electron-electron

scattering and the phonon-electron interaction which affect the $\hat{C}_{ij}^{nm}$ matrix elements in an irreversible way and produce dephasing.

We compute the commutator in the above equation using the anticommutation rules for fermions

$$\begin{aligned}\left\{\hat{c}_i^{n\dagger}, \hat{c}_j^m\right\} &= \delta_{ij}^{nm},\\ \left\{\hat{c}_i^{n\dagger}, \hat{c}_j^{m\dagger}\right\} &= \left\{\hat{c}_i^n, \hat{c}_j^m\right\} = 0,\end{aligned} \tag{7}$$

from which we obtain

$$\left[\hat{C}_{ij}^{nm}, \hat{C}_{kl}^{st}\right] = \delta_{jk}^{ms}\hat{C}_{il}^{nt} - \delta_{il}^{nt}\hat{C}_{kj}^{sm}. \tag{8}$$

The z-dependent envelope functions $F_n(z)$ and the corresponding eigenvalues $\hbar\omega_n$ are obtained from the one-dimensional Schrödinger equation along the growth direction

$$\left[-\frac{\hbar^2}{2}\frac{\mathrm{d}}{\mathrm{d}z}\frac{1}{m_{ez}}\frac{\mathrm{d}}{\mathrm{d}z} + V_{\mathrm{conf}}(z)\right]F_n(z) = \hbar\omega_n F_n(z), \tag{9}$$

where the mass m_{ez} is different for the well and the barrier, and the confining potential profile $V_{\mathrm{conf}}(z)$ is indicated in Figure 1. In the following we only consider the z-components of the electric field and of the dipole moment, since it is the only component which allows intersubband transitions [24]. Next we use for the operators $\hat{c}_i^n$ and $\hat{c}_i^{n\dagger}$ the continuous representations $\hat{c}^n(\boldsymbol{\rho})$ and $\hat{c}^{n\dagger}(\boldsymbol{\rho})$ in the QW planes and substitute in equation (6) the operators by their mean values, so that, using the notation

$$\begin{aligned}C_{12}^{nm} &\equiv C^{nm}(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \left\langle \hat{c}^{n\dagger}(\boldsymbol{\rho}_1)\hat{c}^m(\boldsymbol{\rho}_2)\right\rangle,\\ C_{j\rho}^{nm} &\equiv C^{nm}(\boldsymbol{\rho}_j, \boldsymbol{\rho}) = \left\langle \hat{c}^{n\dagger}(\boldsymbol{\rho}_j)\hat{c}^m(\boldsymbol{\rho})\right\rangle,\\ M_{j\rho}^{nm} &= M_0^{nm}\delta(\boldsymbol{\rho}_j - \boldsymbol{\rho}),\\ M_0^{nm} &= \mathrm{e}\int F_n^*(z) z F_m(z)\mathrm{d}z,\\ E_{j\rho}^1 &= E^1\left(\frac{\boldsymbol{\rho}_j + \boldsymbol{\rho}}{2}\right),\\ E_{j\rho}^2 &= E^2\left(\frac{\boldsymbol{\rho}_j + \boldsymbol{\rho}}{2}\right),\end{aligned} \tag{10}$$

where $j = 1, 2$, we obtain the following equations of motion for the matrices C_{12}^{nm}

$$\begin{aligned}\frac{\partial C_{12}^{11}}{\partial t} + \frac{\mathrm{i}}{\hbar}H_{12}^0 C_{12}^{11} = -\frac{\mathrm{i}}{\hbar}\Bigg[&\int M_{1\boldsymbol{\rho}}^{31}E_{1\boldsymbol{\rho}}^1 C_{\boldsymbol{\rho}2}^{31}d^2\boldsymbol{\rho}\\ &- \int M_{2\boldsymbol{\rho}}^{13}E_{2\boldsymbol{\rho}}^1 C_{1\boldsymbol{\rho}}^{13}d^2\boldsymbol{\rho}\Bigg] + \left(\frac{\mathrm{d}C_{12}^{11}}{\mathrm{d}t}\right)_{\mathrm{incoh}},\end{aligned} \tag{11}$$

$$\begin{aligned}\frac{\partial C_{12}^{22}}{\partial t} + \frac{\mathrm{i}}{\hbar}H_{12}^0 C_{12}^{22} = -\frac{\mathrm{i}}{\hbar}\Bigg[&\int M_{1\boldsymbol{\rho}}^{32}E_{1\boldsymbol{\rho}}^2 C_{\boldsymbol{\rho}2}^{32}d^2\boldsymbol{\rho}\\ &- \int M_{2\boldsymbol{\rho}}^{23}E_{2\boldsymbol{\rho}}^2 C_{1\boldsymbol{\rho}}^{23}d^2\boldsymbol{\rho}\Bigg] + \left(\frac{\mathrm{d}C_{12}^{22}}{\mathrm{d}t}\right)_{\mathrm{incoh}},\end{aligned} \tag{12}$$

$$\begin{aligned}\frac{\partial C_{12}^{33}}{\partial t} + \frac{\mathrm{i}}{\hbar}H_{12}^0 C_{12}^{33} = -\frac{\mathrm{i}}{\hbar}\Bigg[&-\int M_{2\boldsymbol{\rho}}^{31}E_{2\boldsymbol{\rho}}^1 C_{1\boldsymbol{\rho}}^{31}d^2\boldsymbol{\rho}\\ &+ \int M_{1\boldsymbol{\rho}}^{13}E_{1\boldsymbol{\rho}}^1 C_{\boldsymbol{\rho}2}^{13}d^2\boldsymbol{\rho} - \int M_{2\boldsymbol{\rho}}^{32}E_{2\boldsymbol{\rho}}^2 C_{1\boldsymbol{\rho}}^{32}d^2\boldsymbol{\rho}\\ &+ \int M_{1\boldsymbol{\rho}}^{23}E_{1\boldsymbol{\rho}}^2 C_{\boldsymbol{\rho}2}^{23}d^2\boldsymbol{\rho}\Bigg] + \left(\frac{\mathrm{d}C_{12}^{33}}{\mathrm{d}t}\right)_{\mathrm{incoh}},\end{aligned} \tag{13}$$

$$\begin{aligned}\frac{\partial C_{12}^{13}}{\partial t} + \frac{\mathrm{i}}{\hbar}H_{12}^0 C_{12}^{13} - \mathrm{i}(\omega_1 - \omega_3)C_{12}^{13} =\\ -\frac{\mathrm{i}}{\hbar}\Bigg[-\int M_{1\boldsymbol{\rho}}^{31}E_{1\boldsymbol{\rho}}^1 C_{\boldsymbol{\rho}2}^{11}d^2\boldsymbol{\rho} + \int M_{2\boldsymbol{\rho}}^{31}E_{2\boldsymbol{\rho}}^1 C_{1\boldsymbol{\rho}}^{33}d^2\boldsymbol{\rho}\\ - \int M_{2\boldsymbol{\rho}}^{32}E_{2\boldsymbol{\rho}}^2 C_{1\boldsymbol{\rho}}^{12}d^2\boldsymbol{\rho}\Bigg] + \left(\frac{\mathrm{d}C_{12}^{31}}{\mathrm{d}t}\right)_{\mathrm{incoh}}\end{aligned} \tag{14}$$

$$\begin{aligned}\frac{\partial C_{12}^{23}}{\partial t} + \frac{\mathrm{i}}{\hbar}H_{12}^0 C_{12}^{23} - \mathrm{i}(\omega_2 - \omega_3)C_{12}^{23} =\\ -\frac{\mathrm{i}}{\hbar}\Bigg[-\int M_{1\boldsymbol{\rho}}^{32}E_{1\boldsymbol{\rho}}^2 C_{\boldsymbol{\rho}2}^{22}d^2\boldsymbol{\rho} + \int M_{2\boldsymbol{\rho}}^{32}E_{2\boldsymbol{\rho}}^2 C_{1\boldsymbol{\rho}}^{33}d^2\boldsymbol{\rho}\\ - \int M_{1\boldsymbol{\rho}}^{31}E_{1\boldsymbol{\rho}}^1 C_{\boldsymbol{\rho}2}^{21}d^2\boldsymbol{\rho}\Bigg] + \left(\frac{\mathrm{d}C_{12}^{32}}{\mathrm{d}t}\right)_{\mathrm{incoh}},\end{aligned} \tag{15}$$

$$\begin{aligned}\frac{\partial C_{12}^{12}}{\partial t} + \frac{\mathrm{i}}{\hbar}H_{12}^0 C_{12}^{12} - \mathrm{i}(\omega_1 - \omega_2)C_{12}^{12} =\\ -\frac{\mathrm{i}}{\hbar}\left[-\int M_{1\boldsymbol{\rho}}^{23}E_{1\boldsymbol{\rho}}^2 C_{\boldsymbol{\rho}2}^{13}d^2\boldsymbol{\rho} + \int M_{2\boldsymbol{\rho}}^{31}E_{2\boldsymbol{\rho}}^1 C_{1\boldsymbol{\rho}}^{32}d^2\boldsymbol{\rho}\right]\\ + \left(\frac{\mathrm{d}C_{12}^{21}}{\mathrm{d}t}\right)_{\mathrm{incoh}},\end{aligned} \tag{16}$$

where the term H^0 is derived from the mobility term H_{M} [18] and gives

$$H_{12}^0 = \frac{\hbar^2}{2m_{e\|}}\left(\boldsymbol{\nabla}_{\boldsymbol{\rho}_1}^2 - \boldsymbol{\nabla}_{\boldsymbol{\rho}_2}^2\right), \tag{17}$$

assuming a parabolic band shape with an isotropic effective electron mass $m_{e\|} = m_e^*$. Analogous equations hold for the hermitian conjugate operators $C_{21}^{31}, C_{21}^{32}, C_{21}^{21}$. In comparing the above expressions with the usual density matrix equations for three-level atoms, we observe that the difference originates from the fact that in our case

the matrix elements are space dependent, and so are all processes involved. As discussed above, consistently with the approximate Hamiltonian (2), the above coupled equations at this stage do not include energy renormalization effects and the corrections on the Rabi terms discussed by Haug and Koch [22].

The next step is to solve the constitutive equations (11–16), with the electric field in the form (5). The transition density matrix elements C^{13}, C^{12}, C^{23} and their hermitian conjugates can be expressed in the form

$$\begin{aligned}
&C_{12}^{11} = \tilde{C}_{12}^{11}, \quad C_{12}^{22} = \tilde{C}_{12}^{22}, \quad C_{12}^{33} = \tilde{C}_{12}^{33} \\
&C_{12}^{13} = \tilde{C}_{12}^{13}\mathrm{e}^{-\mathrm{i}\Omega_1 t}, \quad C_{12}^{31} = \tilde{C}_{12}^{31}\mathrm{e}^{+\mathrm{i}\Omega_1 t}, \\
&C_{12}^{23} = \tilde{C}_{12}^{23}\mathrm{e}^{-\mathrm{i}\Omega_2 t}, \quad C_{12}^{32} = \tilde{C}_{12}^{32}\mathrm{e}^{+\mathrm{i}\Omega_2 t}, \\
&C_{12}^{12} = \tilde{C}_{12}^{12}\mathrm{e}^{-\mathrm{i}(\Omega_1-\Omega_2)t}, \\
&C_{12}^{21} = \tilde{C}_{12}^{21}\mathrm{e}^{+\mathrm{i}(\Omega_1-\Omega_2)t},
\end{aligned} \tag{18}$$

where the fast time dependence with the frequencies of the e.m. fields is separated out. Introducing the above expressions into the r.h.s. of the equations (11–16) and integrating in $d^2\boldsymbol{\rho}$ we obtain the following equations for the slowly varying parts and for the diagonal population terms

$$\frac{\partial \tilde{C}_{12}^{11}}{\partial t} + \mathrm{i}H_{12}^0\tilde{C}_{12}^{11} = -\frac{\mathrm{i}}{2\hbar}\left[M_0^{31}\tilde{E}_{11}^1\tilde{C}_{12}^{31} - M_0^{13}\tilde{E}_{22}^{1*}\tilde{C}_{12}^{13}\right] + \left(\frac{\mathrm{d}\tilde{C}_{12}^{11}}{\mathrm{d}t}\right)_{\mathrm{incoh}}, \tag{19}$$

$$\frac{\partial \tilde{C}_{12}^{22}}{\partial t} + \mathrm{i}H_{12}^0\tilde{C}_{12}^{22} = -\frac{\mathrm{i}}{2\hbar}\left[M_0^{32}\tilde{E}_{11}^2\tilde{C}_{12}^{32} - M_0^{23}\tilde{E}_{22}^{2*}\tilde{C}_{12}^{23}\right] + \left(\frac{\mathrm{d}\tilde{C}_{12}^{22}}{\mathrm{d}t}\right)_{\mathrm{incoh}}, \tag{20}$$

$$\frac{\partial \tilde{C}_{12}^{33}}{\partial t} + \mathrm{i}H_{12}^0\tilde{C}_{12}^{33} = -\frac{\mathrm{i}}{2\hbar}\left[-M_0^{31}\tilde{E}_{22}^1\tilde{C}_{12}^{31} + M_0^{13}\tilde{E}_{11}^{1*}\tilde{C}_{12}^{13} - M_0^{32}\tilde{E}_{22}^2\tilde{C}_{12}^{32} + M_0^{23}\tilde{E}_{11}^{2*}\tilde{C}_{12}^{23}\right] + \left(\frac{\mathrm{d}\tilde{C}_{12}^{33}}{\mathrm{d}t}\right)_{\mathrm{incoh}}, \tag{21}$$

$$\frac{\partial \tilde{C}_{12}^{13}}{\partial t} + \mathrm{i}H_{12}^0\tilde{C}_{12}^{13} - \mathrm{i}(\Omega_1-\omega_{31})\tilde{C}_{12}^{13} = -\frac{\mathrm{i}}{2\hbar}\left[-M_0^{31}\tilde{E}_{11}^1 C_{12}^{11} + M_0^{31}\tilde{E}_{22}^1 C_{12}^{33} - M_0^{32}\tilde{E}_{22}^2\tilde{C}_{12}^{12}\right] + \left(\frac{\mathrm{d}\tilde{C}_{12}^{31}}{\mathrm{d}t}\right)_{\mathrm{incoh}}, \tag{22}$$

$$\frac{\partial \tilde{C}_{12}^{23}}{\partial t} + \mathrm{i}H_{12}^0\tilde{C}_{12}^{23} - \mathrm{i}(\Omega_2-\omega_{32})\tilde{C}_{12}^{23} = -\frac{\mathrm{i}}{2\hbar}\left[-M_0^{32}\tilde{E}_{11}^2 C_{12}^{22} + M_0^{32}\tilde{E}_{22}^2 C_{12}^{33} - M_0^{31}\tilde{E}_{11}^1\tilde{C}_{12}^{21}\right] + \left(\frac{\mathrm{d}\tilde{C}_{12}^{32}}{\mathrm{d}t}\right)_{\mathrm{incoh}}, \tag{23}$$

$$\frac{\partial \tilde{C}_{12}^{12}}{\partial t} + \mathrm{i}H_{12}^0\tilde{C}_{12}^{12} - \mathrm{i}(\Omega_1-\Omega_2-\omega_{21})\tilde{C}_{12}^{12} = -\frac{\mathrm{i}}{2\hbar}\left[-M_0^{23}\tilde{E}_{11}^{2*}\tilde{C}_{12}^{13} + M_0^{31}\tilde{E}_{22}^1\tilde{C}_{12}^{32}\right] + \left(\frac{\mathrm{d}\tilde{C}_{12}^{21}}{\mathrm{d}t}\right)_{\mathrm{incoh}}, \tag{24}$$

where $\omega_{ij} = \omega_i - \omega_j$.

Now we make the approximation that the wavelength of the electromagnetic wave is much greater than the lateral extension of the considered QW, which is certainly valid for the infrared region of the spectrum. Thus the field can be considered as uniform, and the electron density matrix elements will depend on the relative coordinate only, and can be expressed in the two dimensional space as

$$\tilde{C}_{12}^{ij} = \sum_{\mathbf{k}} \tilde{c}_{\mathbf{k}}^{ij}\mathrm{e}^{-\mathrm{i}\mathbf{k}(\boldsymbol{\rho}_1-\boldsymbol{\rho}_2)}. \tag{25}$$

Our objective is to solve the equations for the Fourier components $\tilde{c}_{\mathbf{k}}^{ij}$, which have a slow time dependence, in the two dimensional momentum space; this requires putting expansion (25) into the constitutive equations (19–24) and solving them for all the values of the vector $\mathbf{k}$. The coefficients $\tilde{c}_{\mathbf{k}}^{ij}$ can also be defined from the electron creation (destruction) operators in k space $\hat{c}_{\mathbf{k}}^{i\dagger}$ ($\hat{c}_{\mathbf{k}}^{i}$). They are the slowly varying parts of their mean values

$$c_{\mathbf{k}}^{ij} = \left\langle \hat{c}_{\mathbf{k}}^{i\dagger}\hat{c}_{\mathbf{k}}^{j} \right\rangle. \tag{26}$$

so that

$$\begin{aligned}
&c_{\mathbf{k}}^{11} = \tilde{c}_{\mathbf{k}}^{11}, \quad c_{\mathbf{k}}^{22} = \tilde{c}_{\mathbf{k}}^{22}, \quad c_{\mathbf{k}}^{33} = \tilde{c}_{\mathbf{k}}^{3}, \\
&c_{\mathbf{k}}^{13} = \tilde{c}_{\mathbf{k}}^{13}\mathrm{e}^{-\mathrm{i}\Omega_1 t}, \quad c_{\mathbf{k}}^{31} = \tilde{c}_{\mathbf{k}}^{31}\mathrm{e}^{+\mathrm{i}\Omega_1 t}, \\
&c_{\mathbf{k}}^{23} = \tilde{c}_{\mathbf{k}}^{23}\mathrm{e}^{-\mathrm{i}\Omega_2 t}, \quad c_{\mathbf{k}}^{32} = \tilde{c}_{\mathbf{k}}^{32}\mathrm{e}^{+\mathrm{i}\Omega_2 t}, \\
&c_{\mathbf{k}}^{12} = \tilde{c}_{\mathbf{k}}^{12}\mathrm{e}^{-\mathrm{i}(\Omega_1-\Omega_2)t}, \\
&c_{\mathbf{k}}^{21} = \tilde{c}_{\mathbf{k}}^{21}\mathrm{e}^{+\mathrm{i}(\Omega_1-\Omega_2)t}.
\end{aligned} \tag{27}$$

A crucial point is to compute the incoherent contributions, which are typical of the quantum well system and are crucial for the possibility of obtaining EIT, as discussed by Sadeghi *et al.* [5].

3 The dephasing terms

We now consider the incoherent irreversible terms on the r.h.s. of the equations (19–24). They are essentially due to the electron-electron and electron-phonon scattering, because the spontaneous radiative emission, which is dominant in the atomic case, gives here a negligible contribution.

3.1 Electron-electron scattering

We consider the main electron-electron scattering contributions to the incoherent terms produced by the additional interaction Hamiltonian term which conserves the number of electrons in each subband [16]

$$H_{e-e} = \frac{1}{2} \sum_{i,j,\mathbf{k},\mathbf{k'},\mathbf{q}\neq 0} V_{ij}^{e-e}(\mathbf{q}) \hat{c}_{\mathbf{k}+\mathbf{q}}^{i\dagger} \hat{c}_{\mathbf{k'}-\mathbf{q}}^{j\dagger} \hat{c}_{\mathbf{k'}}^{j} \hat{c}_{\mathbf{k}}^{i} \quad . \tag{28}$$

The additional contributions which scatter electrons between different subbands are negligible. We note that the inclusion of the Hamiltonian term (28), introduces into the equations of motion four-operator terms. In the Hartree-Fock approximation these terms can be written as the product of appropriate averages of two-operator terms, leading to the already discussed corrections to the subband dispersion and to the Rabi frequencies. All the higher order correlations can be systematically included to the desired order in the Coulomb potentials V_{ij}^{e-e} by the use of the projection operator technique [16]; this procedure allows to separate the equations of motion into two parts, the coherent Hartree-Fock terms and incoherent or collisional ones. This section will be concerned with the incoherent contributions in the rate equations.

The estimation of the population change in each subband leads to the well-known semiclassical Boltzmann expression [22]

$$\left(\frac{\partial c_{\mathbf{k}}^{ii}}{\partial t}\right)_{\text{incoh}}^{e-e} = \Gamma_i^{\text{in,e}}(\mathbf{k})(1-c_{\mathbf{k}}^{ii}) - \Gamma_i^{\text{out,e}}(\mathbf{k}) c_{\mathbf{k}}^{ii}, \tag{29}$$

and the polarization dephasing rates can be written as [22]

$$\begin{aligned}\left(\frac{\partial c_{\mathbf{k}}^{ij}}{\partial t}\right)_{\text{incoh}}^{e-e} = &-\frac{1}{2}\Big[\Gamma_i^{\text{in,e}}(\mathbf{k})\Gamma_i^{\text{out,e}}(\mathbf{k}) \\ &+\Gamma_j^{\text{in,e}}(\mathbf{k}) + \Gamma_j^{\text{out,e}}(\mathbf{k})\Big] c_{\mathbf{k}}^{ij} \\ &+ \sum_{\mathbf{q}\neq 0} \Lambda_{ij}(\mathbf{k},\mathbf{q}) c_{\mathbf{k}+\mathbf{q}}^{ij} \quad i \neq j, \end{aligned} \tag{30}$$

expressed in terms of the probability rates for the electrons to be scattered in or out of a state $\mathbf{k}$ of subband i

$$\begin{aligned}\Gamma_i^{\text{in,e}}(\mathbf{k}) = &\frac{2\pi}{\hbar} 2 \sum_{j,\mathbf{q},\mathbf{k'}} c_{\mathbf{k}-\mathbf{q}}^{ii}(1-c_{\mathbf{k'}-\mathbf{q}}^{jj}) c_{\mathbf{k'}}^{jj} |V_{ij}^{e-e}(\mathbf{q})|^2 \delta \\ &\times \left[E_j^c(\mathbf{k'}-\mathbf{q}) - E_j^c(\mathbf{k'}) - E_i^c(\mathbf{k}-\mathbf{q}) + E_i^c(\mathbf{k})\right], \end{aligned} \tag{31}$$

$$\begin{aligned}\Gamma_i^{\text{out,e}}(\mathbf{k}) = &\frac{2\pi}{\hbar} 2 \sum_{j,\mathbf{q},\mathbf{k'}} (1-c_{\mathbf{k}-\mathbf{q}}^{ii}) c_{\mathbf{k'}-\mathbf{q}}^{jj}(1-c_{\mathbf{k'}}^{jj}) \\ &\times |V_{ij}^{e-e}(\mathbf{q})|^2 \delta \left[E_j^c(\mathbf{k'}-\mathbf{q})\right. \\ &\left. -E_j^c(\mathbf{k'}) - E_i^c(\mathbf{k}-\mathbf{q}) + E_i^c(\mathbf{k})\right], \end{aligned} \tag{32}$$

and of the polarization transfer contributions of Lindberg and Koch [16]

$$\begin{aligned}&\Lambda_{ij}(\mathbf{k},\mathbf{q}) = \\ &\sum_{\mathbf{k'}\neq 0, n=i,j} \frac{2\pi}{\hbar} 2 \left[c_{\mathbf{k}}^{nn}(1-c_{\mathbf{k'}}^{nn}) c_{\mathbf{k'}+\mathbf{q}}^{nn} + (1-c_{\mathbf{k}}^{nn}) c_{\mathbf{k'}}^{nn}(1-c_{\mathbf{k'}+\mathbf{q}}^{nn})\right] \\ &\times |V_{nn}^{e-e}(\mathbf{q})|^2 \delta \left[E_n^c(\mathbf{k'}+\mathbf{q}) + E_n^c(\mathbf{k}) - E_n^c(\mathbf{k'}) - E_n^c(\mathbf{k}+\mathbf{q})\right] \\ &+ \sum_{\mathbf{k'}\neq 0, (n,m)=\{(i,j),(j,i)\}} \frac{2\pi}{\hbar} 2 \left[c_{\mathbf{k}}^{nn}(1-c_{\mathbf{k'}}^{mm}) c_{\mathbf{k'}-\mathbf{q}}^{mm}\right. \\ &\left. +(1-c_{\mathbf{k}}^{nn}) c_{\mathbf{k'}-\mathbf{q}}^{mm}(1-c_{\mathbf{k'}-\mathbf{q}}^{mm})\right] |V_{nm}^{e-e}(\mathbf{q})|^2 \delta \left[E_n^c(\mathbf{k})\right. \\ &\left. +E_m^c(\mathbf{k'}-\mathbf{q}) - E_n^c(\mathbf{k}+\mathbf{q}) - E_m^c(\mathbf{k'})\right], \end{aligned} \tag{33}$$

where all vectors are two-dimensional and the factor 2 accounts for the sum over spins. The term V_{ij}^{e-e} denotes the screened static electron-electron interaction potential, which in two dimensions takes the form

$$V_{ij}^{e-e}(q) = \frac{2\pi e^2}{Sq} \frac{F_{ij}(q)}{\epsilon_{ij}(q)}, \tag{34}$$

S being the QW surface, $F_{ij}(q)$ the form factor which takes into account confinement in the z-direction

$$F_{ij}(q) = \int \mathrm{d}z \int \mathrm{d}z_1 \left|F_i(z)\right|^2 \left|F_j(z_1)\right|^2 \mathrm{e}^{-q|z-z_1|}, \tag{35}$$

and $\epsilon_{ij}(q)$ being the static dielectric functions, which we obtained in the long wavelength limit by solving an appropriate set of equations given by Lee and Galbraith [25].

We can evaluate the summations in equations (31, 32) by transforming them into integrations and obtain

$$\begin{aligned}\Gamma_i^{\text{out,e}}(k) = &\sum_j 2 \left(\frac{S}{4\pi^2}\right)^2 \frac{4\pi}{\hbar} \int_0^\infty \mathrm{d}q \int_0^{2\pi} \mathrm{d}\theta_q \int_{|k\cos\theta_q|}^\infty \mathrm{d}k' \\ &\times c_{\mathbf{k'}-\mathbf{q}}^{jj}(1-c_{\mathbf{k}-\mathbf{q}}^{ii})(1-c_{\mathbf{k'}}^{jj}) |V_{ij}^{e-e}(q)|^2 \\ &\times \frac{m_e^*}{\hbar^2 \sqrt{1-(k/k')^2\cos^2\theta_q}}. \end{aligned} \tag{36}$$

A similar calculation can be performed to obtain $\Gamma_i^{\text{in,e}}(\mathbf{k})$ from expression (31) and both dephasing rates can be computed by numerical integration.

To evaluate the last term of equation (30) we find it convenient to transform it as

$$\sum_{\mathbf{q}\neq 0} \Lambda_{ij}(\mathbf{k},\mathbf{q}) c_{\mathbf{k}+\mathbf{q}}^{ij} = \sum_{n=i,j} \Lambda_{ij,n}^{\text{out}}(\mathbf{k}) c_{\mathbf{k}}^{nn} + \Lambda_{ij,n}^{\text{in}}(\mathbf{k}) (1-c_{\mathbf{k}}^{nn}), \tag{37}$$

with

$$\Lambda^{\rm in}_{ij,n}(\mathbf{k}) = \sum_{\mathbf{k}'\neq 0,\mathbf{q}\neq 0} \frac{2\pi}{\hbar} 2\left[c^{ij}_{\mathbf{k}+\mathbf{q}} c^{nn}_{\mathbf{k}'}(1-c^{nn}_{\mathbf{k}'+\mathbf{q}})\right] \times |V^{e-e}_{nn}(\mathbf{q})|^2 \delta\left[E^c_n(\mathbf{k}'+\mathbf{q}) + E^c_n(\mathbf{k}) - E^c_n(\mathbf{k}') - E^c_n(\mathbf{k}+\mathbf{q})\right] + \sum_{\mathbf{k}'\neq 0,\mathbf{q}\neq 0,m=\{i,j\}\neq n} \frac{2\pi}{\hbar} \times 2\left[c^{ij}_{\mathbf{k}+\mathbf{q}} c^{mm}_{\mathbf{k}'}(1-c^{mm}_{\mathbf{k}'-\mathbf{q}})\right] \times |V^{e-e}_{nm}(\mathbf{q})|^2 \delta\left[E^c_n(\mathbf{k}) + E^c_m(\mathbf{k}'-\mathbf{q}) - E^c_n(\mathbf{k}+\mathbf{q}) - E^c_m(\mathbf{k}')\right]. \quad (38)$$

$$\Lambda^{\rm out}_{ij,n}(\mathbf{k}) = \sum_{\mathbf{k}'\neq 0,\mathbf{q}\neq 0} \frac{2\pi}{\hbar} 2\left[c^{ij}_{\mathbf{k}+\mathbf{q}}(1-c^{nn}_{\mathbf{k}'}) c^{nn}_{\mathbf{k}'+\mathbf{q}}\right] \times |V^{e-e}_{nn}(\mathbf{q})|^2 \delta\left[E^c_n(\mathbf{k}'+\mathbf{q}) + E^c_n(\mathbf{k}) - E^c_n(\mathbf{k}') - E^c_n(\mathbf{k}+\mathbf{q})\right] + \sum_{\mathbf{k}'\neq 0,\mathbf{q}\neq 0,m=\{i,j\}\neq n} \frac{2\pi}{\hbar} \times 2\left[c^{ij}_{\mathbf{k}+\mathbf{q}}(1-c^{mm}_{\mathbf{k}'}) c^{mm}_{\mathbf{k}'-\mathbf{q}}\right] \times |V^{e-e}_{nm}(\mathbf{q})|^2 \delta\left[E^c_n(\mathbf{k}) + E^c_m(\mathbf{k}'-\mathbf{q}) - E^c_n(\mathbf{k}+\mathbf{q}) - E^c_m(\mathbf{k}')\right]. \quad (39)$$

Expressions (38) and (39) can be numerically computed by the same approach described above for $\Gamma^{\rm in,e}_i(\mathbf{k})$ and $\Gamma^{\rm out,e}_i(\mathbf{k})$.

3.2 Electron-phonon scattering

To estimate the incoherent part of the constitutive equations we must also consider the electron-phonon effects, which produce intersubband and intrasubband scattering. Since we intend to consider QW in III-V compounds, we take into account only LO phonons (frequency $\omega_{\rm LO}$) because their coupling constants are much larger than those of acoustic phonons. In fact, writing the phonon-electron Hamiltonian in terms of the phonon creation (destruction) operators $\hat{b}^\dagger_{\mathbf{q}}$ ($\hat{b}_{\mathbf{q}}$)

$$H_{\rm e-p} = \sum_{\mathbf{q}} \left[\alpha(\mathbf{q}) {\rm e}^{-{\rm i}\mathbf{q}\cdot\mathbf{r}} \hat{b}^\dagger_{\mathbf{q}} + {\rm c.c.}\right], \quad (40)$$

we have for the two cases, as shown by Ferreira and Bastard [26],

$$|\alpha_{\rm LO}(\mathbf{q})|^2 = 2\pi\hbar\omega_{\rm LO}\frac{e^2}{\epsilon^* \mathcal{V} q^2},$$
$$|\alpha_{ac}(\mathbf{q})|^2 = \frac{C_0}{\mathcal{V}}\hbar c_s q, \quad (41)$$

where $\mathcal{V}$ is the volume, c_s the sound velocity in the crystal, C_0 a constant depending on the density of the material and ϵ^* is expressed in terms of the dielectric function at high frequency ϵ_∞ and static ϵ_s as

$$\frac{1}{\epsilon^*} = \frac{1}{\epsilon_\infty} - \frac{1}{\epsilon_s}\,. \quad (42)$$

This gives for the densities of interest a relation between orders of magnitude which can be computed at the Fermi level as

$$\frac{|\alpha_{\rm LO}(\mathbf{q}_F)|^2}{|\alpha_{ac}(\mathbf{q}_F)|^2} \approx 10^7. \quad (43)$$

For convenience we perform a low temperature calculation, since in the case of nanostructures the temperature can be fixed at will. Consequently, we consider only the possibility of phonon emission with electrons decaying from higher subbands into the lower subbands and within the same band. The transition probabilities from band i to band j for an electron with initial wave vector $\mathbf{k}$, the initial state being taken occupied and the final state empty, are

$$\Gamma^p_{ij}(\mathbf{k}) = \frac{2\pi}{\hbar} 2 \sum_{\mathbf{q}} (N_{\rm p}+1) \times |V^{\rm e-op}_{ij}(\mathbf{q})|^2 \delta\left[E^c_j(\mathbf{k}+\mathbf{q}) + \hbar\omega_{\rm LO} - E^c_i(\mathbf{k})\right], \quad (44)$$

where the phonon distribution is

$$N_p = \left[\exp\left(\frac{\hbar\omega_{\rm LO}}{kT}\right) - 1\right]^{-1}, \quad (45)$$

and $|V^{\rm e-op}_{ij}(\mathbf{q})|^2$ is the 2D interaction potential, given explicitly by

$$|V^{\rm e-op}_{ij}(\mathbf{q})|^2 = \frac{\pi e^2 \hbar\omega_{\rm LO}}{2S\epsilon^*}\frac{I_{ij}(q)}{q}, \quad (46)$$

where

$$I_{ij}(q) = \int {\rm d}z \int {\rm d}z_1 F_i(z) F^*_j(z_1) {\rm e}^{-q|z-z_1|} F^*_i(z_1) F_j(z), \quad (47)$$

$$q = \sqrt{k^2 + k'^2 - 2kk'\cos\theta}, \quad (48)$$

and

$$k' = \sqrt{k^2 + \frac{2m^*_e(\hbar\omega_{ij} - \hbar\omega_{\rm LO})}{\hbar^2}}, \quad (49)$$

as required by the conservation laws.

To evaluate summation (44) we proceed as for the electron-electron scattering, obtaining a one dimensional integral, of the same type as given in reference [26],

$$\Gamma_{ij}^{p}(k) = \frac{e^2\omega_{\mathrm{LO}}m_e^*}{2\hbar^2\epsilon^*}(N_p+1)\int_0^{2\pi}\frac{I_{ij}(q)}{q}\mathrm{d}\theta. \tag{50}$$

This procedure eliminates the angular dependence of $\mathbf{k}$, so that from now on we only consider the moduli k.

As a consequence we can express the total population change rate in a subband i at a given wave vector k as

$$\left(\frac{\partial c_k^{ii}}{\partial t}\right)_{\mathrm{incoh}}^{\mathrm{e-p}} = -\sum_{j\le i}\Gamma_{ij}^{\mathrm{out,p}}(k)c_k^{ii} + \sum_{j\ge i}\Gamma_{ji}^{\mathrm{out,p}}(k''_{ji})c_{k''_{ji}}^{jj}, \tag{51}$$

where $\Gamma_{ij}^{\mathrm{out,p}}(k)$ is the probability that an electron is scattered by a phonon out of state k from band i to band j

$$\Gamma_{ij}^{\mathrm{out,p}}(k) = \Gamma_{ij}^{p}(k)\left(1-c_{k'}^{jj}\right), \tag{52}$$

k' is given by expression (49), and

$$k''_{ij} = \sqrt{k^2 - \frac{2m_e^*(\hbar\omega_{ij}-\hbar\omega_{\mathrm{LO}})}{\hbar^2}}\,. \tag{53}$$

Electron-phonon scattering also affects the polarization terms which can be written as

$$\left(\frac{\partial c_k^{ij}}{\partial t}\right)_{\mathrm{incoh}}^{\mathrm{e-p}} = -\frac{1}{2}\left[\sum_{n\le i}\Gamma_{in}^{\mathrm{out,p}}(k) + \sum_{n\le j}\Gamma_{jn}^{\mathrm{out,p}}(k)\right]c_k^{ij}, \quad \text{for} \quad i\neq j. \tag{54}$$

With the above procedure we can compute, with simple numerical integration, all the relevant contributions due to phonons to the dephasing, which appear in the populations and in the transition rates. They can be formally expressed in terms of a single $\mathbf{k}$ vector through the single parameter $\Gamma_{ij}^{\mathrm{out,p}}$, as shown in expressions (49–54).

4 Constitutive equations and their solutions

To obtain the equations for the Fourier transforms $\tilde{c}_k^{ij}$ defined in (25), we use the appropriate expressions for the irreversible dephasing terms, neglecting eventual broadening contributions due to fluctuations of the QW widths. We also include the corrections on the Rabi frequencies produced by the intersubband scattering due to H_{e-e} (28); they are obtained in the Hartree–Fock approximation by writing Heisenberg equations and replacing the four-operators terms with appropriate products of the two-operators coefficients $\tilde{c}_k^{ij}$ as showed by Haug and Koch [22]. The final expressions for the slowly varying parts read as follows

$$\begin{aligned}
\frac{\partial\tilde{c}_k^{11}}{\partial t} &= -\mathrm{i}(\alpha'_k\tilde{c}_k^{31} - \alpha'^*_k\tilde{c}_k^{13}) + \mathrm{i}(\gamma'^*_k\tilde{c}_k^{12} - \gamma'_k\tilde{c}_k^{21}) \\
&\quad + \Gamma_1^{\mathrm{in,e}}(k)\left(1-\tilde{c}_k^{11}\right) - \left[\Gamma_1^{\mathrm{out,e}}(k) + \Gamma_{11}^{\mathrm{out,p}}(k)\right]\tilde{c}_k^{11} \\
&\quad + \Gamma_{21}^{\mathrm{out,p}}(k''_{21})\tilde{c}_{k''_{21}}^{22} + \Gamma_{11}^{\mathrm{out,p}}(k''_{11})\tilde{c}_{k''_{11}}^{11}, \\
\frac{\partial\tilde{c}_k^{22}}{\partial t} &= -\mathrm{i}\left(\beta'_k\tilde{c}_k^{32} - \beta'^*_k\tilde{c}_k^{23}\right) + \mathrm{i}\left(\gamma'_k\tilde{c}_k^{21} - \gamma'^*_k\tilde{c}_k^{12}\right) \\
&\quad + \Gamma_2^{\mathrm{in,e}}(k)\left(1-\tilde{c}_k^{22}\right) - \left[\Gamma_2^{\mathrm{out,e}}(k) + \Gamma_{22}^{\mathrm{out,p}}(k)\right. \\
&\quad \left.+\Gamma_{21}^{\mathrm{out,p}}(k)\right]\tilde{c}_k^{22} + \Gamma_{22}^{\mathrm{out,p}}(k''_{22})\tilde{c}_{k''_{22}}^{22} + \Gamma_{32}^{\mathrm{out,p}}(k''_{32})\tilde{c}_{k''_{32}}^{33}, \\
\frac{\partial\tilde{c}_k^{33}}{\partial t} &= \mathrm{i}\left(\alpha'_k\tilde{c}_k^{31} - \alpha'^*_k\tilde{c}_k^{13} + \beta'_k\tilde{c}_k^{32} - \beta'^*_k\tilde{c}_k^{23}\right) \\
&\quad + \Gamma_3^{\mathrm{in,e}}(k)\left(1-\tilde{c}_k^{33}\right) + \Gamma_{33}^{\mathrm{out,p}}(k''_{33})\tilde{c}_{k''_{33}}^{33} \\
&\quad - \left[\Gamma_3^{\mathrm{out,e}}(k) + \Gamma_{33}^{\mathrm{out,p}}(k) + \Gamma_{32}^{\mathrm{out,p}}(k)\right]\tilde{c}_k^{33}, \\
\frac{\partial\tilde{c}_k^{13}}{\partial t} &- \mathrm{i}\left\{\Delta + \mathrm{i}\left[\Gamma_1^{\mathrm{e}}(k) + \Gamma_1^{\mathrm{p}}(k) + \Gamma_3^{\mathrm{e}}(k) + \Gamma_3^{\mathrm{p}}(k)\right]\right\}\tilde{c}_k^{13} = \\
&\quad \sum_{n=\{1,3\}}\left[\Lambda_{13,n}^{\mathrm{out}}(k)\tilde{c}_k^{nn} + \Lambda_{13,n}^{\mathrm{in}}(k)(1-\tilde{c}_k^{nn})\right] \\
&\quad + \mathrm{i}\alpha'_k\left(\tilde{c}_k^{33} - \tilde{c}_k^{11}\right) - \mathrm{i}\beta'_k\tilde{c}_k^{12} - \mathrm{i}\gamma'_k\tilde{c}_k^{23}, \\
\frac{\partial\tilde{c}_k^{23}}{\partial t} &- \mathrm{i}\left\{\left[\Delta' + \mathrm{i}\left[\Gamma_2^{\mathrm{e}}(k)+\Gamma_2^{\mathrm{p}}(k)+\Gamma_3^{\mathrm{e}}(k)+\Gamma_3^{\mathrm{p}}(k)\right]\right\}\tilde{c}_k^{23} = \\
&\quad \sum_{n=\{2,3\}}\left[\Lambda_{23,n}^{\mathrm{out}}(k)\tilde{c}_k^{nn} + \Lambda_{23,n}^{\mathrm{in}}(k)(1-\tilde{c}_k^{nn})\right] \\
&\quad + \mathrm{i}\beta'_k\left(\tilde{c}_k^{33} - \tilde{c}_k^{22}\right) - \mathrm{i}\alpha'_k\tilde{c}_k^{21} - \mathrm{i}\gamma'^*_k\tilde{c}_k^{13}, \\
\frac{\partial\tilde{c}_k^{12}}{\partial t} &- \mathrm{i}\left\{\Delta - \Delta' + \mathrm{i}\left[\Gamma_1^{\mathrm{e}}(k)+\Gamma_1^{\mathrm{p}}(k)+\Gamma_2^{\mathrm{e}}(k)+\Gamma_2^{\mathrm{p}}(k)\right]\right\}\tilde{c}_k^{12} = \\
&\quad \sum_{n=\{1,2\}}\left[\Lambda_{12,n}^{\mathrm{out}}(k)\tilde{c}_k^{nn} + \Lambda_{12,n}^{\mathrm{in}}(k)(1-\tilde{c}_k^{nn})\right] \\
&\quad - \mathrm{i}\left(\alpha'_k\tilde{c}_k^{32} - \beta'^*_k\tilde{c}_k^{13}\right) + \mathrm{i}\gamma'_k\left(\tilde{c}_k^{11} - \tilde{c}_k^{22}\right),
\end{aligned} \tag{55}$$

where

$$\alpha'_k = \alpha + \frac{1}{2\hbar}\sum_{\mathbf{q}\neq 0}V_{13}(q)\tilde{c}_{\mathbf{k}+\mathbf{q}}^{13}, \quad \alpha = \frac{M_0^{31}\tilde{E}^1}{2\hbar},$$

$$\beta'_k = \beta + \frac{1}{2\hbar}\sum_{\mathbf{q}\neq 0}V_{23}(q)\tilde{c}_{\mathbf{k}+\mathbf{q}}^{23}, \quad \beta = \frac{M_0^{32}\tilde{E}^2}{2\hbar},$$

$$\gamma'_k = \frac{1}{2\hbar}\sum_{\mathbf{q}\neq 0}V_{12}(q)\tilde{c}_{\mathbf{k}+\mathbf{q}}^{12}$$

$$\Delta = \Omega_1 - (\omega_3 - \omega_1) = \Omega_1 - \omega_{31},$$

$$\Delta' = \Omega_2 - (\omega_3 - \omega_2) = \Omega_2 - \omega_{32},$$

$$\Gamma_1^{\mathrm{e}}(k) = \frac{1}{2}\left[\Gamma_1^{\mathrm{in,e}}(k) + \Gamma_1^{\mathrm{out,e}}(k)\right],$$

$$\Gamma_1^{\rm p}(k) = \frac{1}{2}\left[\Gamma_{11}^{\rm out,p}(k)\right],$$

$$\Gamma_2^{\rm e}(k) = \frac{1}{2}\left[\Gamma_2^{\rm in,e}(k) + \Gamma_2^{\rm out,e}(k)\right],$$

$$\Gamma_2^{\rm p}(k) = \frac{1}{2}\left[\Gamma_{22}^{\rm out,p}(k) + \Gamma_{21}^{\rm out,p}(k)\right],$$

$$\Gamma_3^{\rm e}(k) = \frac{1}{2}\left[\Gamma_3^{\rm in,e}(k) + \Gamma_3^{\rm out,e}(k)\right],$$

$$\Gamma_3^{\rm p}(k) = \frac{1}{2}\left[\Gamma_{33}^{\rm out,p}(k)+\Gamma_{31}^{\rm out,p}(k)+\Gamma_{32}^{\rm out,p}(k)\right]. \quad (56)$$

In order to find the equilibrium electron densities $\tilde{c}_k^{ii}$ and coherent transition amplitudes $\tilde{c}_k^{ij}$ $(i \neq j)$ we have to solve the system of equations (55) in the stationary case ($\frac{\partial \tilde{c}_k^{ij}}{\partial t} = 0$) for all k values. We observe that the unknown densities $\tilde{c}_k^{ij}$ enter the scattering rates $\Gamma(k)$ and $\Lambda(k)$, and this produces a mixing between different k values. We notice that conservation of the number of electrons is automatically included for the phonon scattering, but for the electron-electron scattering the number is conserved only by the self-consistency requirement.

Our approach is conceptually similar to that of Sadeghi *et al.* [4], but it differs from it in the fact that we compute explicitly all the dephasing contributions and no experimental parameters are required. All the eventual size fluctuation broadenings can be reduced improving the quality of the sample, and we consider them to be negligible. In our procedure the electron-electron and electron-phonon scattering produce admixture of different k values in the different bands, so that the number of coupled equations is correspondingly increased.

To obtain analytic self-consistent solutions of the nonlinear set of equation (55) is a formidable task. We search for numerical solutions using the following iteration procedure. We first calculate the scattering parameters $\Gamma(k)$ and $\Lambda(k)$, appearing in the equations, in the absence of electromagnetic fields using the Fermi electron distribution law. The values so obtained are inserted into equations (55), obtaining a linearized system which involves all k values. In the next step we eliminate the mixing between different k values by computing also the terms $\tilde{c}_{k''}^{ii}$ with the electron Fermi distribution; consequently we only have to solve a 9×9 linear system for each value of k. The solutions so obtained are again used for calculating the dephasing and $\tilde{c}_{k''}^{ii}$ terms, which are then inserted into (55) to give new solutions. The process is repeated with the appropriate normalization at every step, until self consistency is achieved for the stationary states. In the calculations we performed, self consistency was rapidly obtained in the case of only one control beam resonant with the $2 \to 3$ transition, while about 10 iterations were required in the configurations with two driving fields of frequencies ω_{31} and ω_{32} or with a single driving field of frequency ω_{31}, because of the appreciable population changes in the latter cases.

5 Absorption spectrum and numerical results for a specific system

We assume that the pump field (5) creates a stationary state of the system, and consider the absorption spectrum of an additional weak probe field

$$E(t) = \frac{1}{2}\left[\mathcal{E}(t) + \mathcal{E}^*(t)\right], \quad (57)$$

with

$$\mathcal{E}(t) = \mathcal{E}_0\, {\rm e}^{-{\rm i}\omega_p t}, \quad (58)$$

at a positive probe frequency ω_p, close to the transition frequency ω_{31}. The absorption coefficient is proportional to the imaginary part of the linear susceptibility. We express the perturbation Hamiltonian in terms of the dipole matrix elements and of the electron creation and annihilation operators [1]

$$\begin{aligned} H' &= -\sum_{\mathbf{k}} \left[M_0^{31}\mathcal{E}(t)\hat{c}_{\mathbf{k}}^{3\dagger}\,\hat{c}_{\mathbf{k}}^{1} + M_0^{13}\mathcal{E}^*(t)\,\hat{c}_{\mathbf{k}}^{1\dagger}\,\hat{c}_{\mathbf{k}}^{3}\right] \\ &= -\sum_{\mathbf{k}} \left[\hat{p}_{\mathbf{k}}^{+}(t)\mathcal{E}(t) + \hat{p}_{\mathbf{k}}^{-}(t)\mathcal{E}^*(t)\right], \end{aligned} \quad (59)$$

where

$$\begin{aligned} \hat{p}_{\mathbf{k}}^{+} &= M_0^{31}\hat{c}_{\mathbf{k}}^{3\dagger}\,\hat{c}_{\mathbf{k}}^{1}, \\ \hat{p}_{\mathbf{k}}^{-} &= M_0^{13}\hat{c}_{\mathbf{k}}^{1\dagger}\,\hat{c}_{\mathbf{k}}^{3}, \end{aligned} \quad (60)$$

are the positive and negative frequency components of the system polarization operators at a specific $\mathbf{k}$.

The derivation of the expression for the imaginary part of the susceptibility, which gives the absorption, is presented in Appendix A. It is given by the Laplace transform of the correlation function

$${\rm Im}\left[\chi(\omega_p)\right] = \frac{1}{\hbar}\sum_{\mathbf{k}} c_{\mathbf{k}}^{11}{\rm Re}\left\{\mathcal{L}[G_{\mathbf{k}}(\tau,t');s={\rm i}\omega_p]\right\}, \quad (61)$$

with

$$\mathcal{L}[G_{\mathbf{k}}(\tau,t');s] = \int_0^\infty G_{\mathbf{k}}(\tau,t'){\rm e}^{-s\tau}{\rm d}\tau, \quad (62)$$

where the correlation function is defined as

$$G_{\mathbf{k}}(\tau,t') = \left\langle\left[\hat{p}_{\mathbf{k}}^{-}(t'),\hat{p}_{\mathbf{k}}^{+}(t'+\tau)\right]\right\rangle. \quad (63)$$

With the use of the quantum regression theorem [30] it is finally possible to evaluate the susceptibility as defined above in terms of the steady state solutions of the equations (55). With the procedure of the Appendix A we obtain the absorption contribution in the form

$$\begin{aligned} {\rm Im}[\chi(\omega_p)] = -\frac{\left|M_0^{13}\right|^2}{\hbar}\sum_{\mathbf{k}}\tilde{c}_{\mathbf{k}}^{11}\ {\rm Re}\Bigg\{&\tilde{c}_{\mathbf{k}}^{13}\left(R_{\mathbf{k},\mathbf{k}}^{31,11}(z)\right. \\ &\left.-R_{\mathbf{k},\mathbf{k}}^{31,33}(z)\right) + \left(\tilde{c}_{\mathbf{k}}^{33}-\tilde{c}_{\mathbf{k}}^{11}\right)R_{\mathbf{k},\mathbf{k}}^{31,31}(z) \\ &+\tilde{c}_{\mathbf{k}}^{23}R_{\mathbf{k},\mathbf{k}}^{31,21}(z) - \tilde{c}_{\mathbf{k}}^{12}R_{\mathbf{k},\mathbf{k}}^{31,32}(z)\Bigg\}, \end{aligned} \quad (64)$$

Table 1. Parameter values for GaAs/$Ga_{1-x}Al_xAs$ Asymmetric Double Quantum Wells from Figure 1. Energies $\hbar\omega_n$ are referred to the bottom of the well conduction band.

Quantity	Value	Quantity	Value
m_e^*	0.0665 m_0	$\hbar\omega_{\rm LO}$	36.4 meV
ϵ_s	12.35	ϵ_∞	10.92
n_e	5×10^{10} cm^{-2}	T	6 K
E_F	3.6 meV	k_F	0.08 nm^{-1}
L_1	1.8 nm	L_2	5.6 nm
d	3 nm	$\hbar\omega_1$	68.5 meV
$\hbar\omega_2$	191.0 meV	$\hbar\omega_3$	263.3 meV

where $z = \mathrm{i}(\omega_p - \Omega_1)$ and the tensor elements $R^{ij,lm}_{\mathbf{k},\mathbf{k}}(z)$ are obtained from the equations (55) written in the matrix form

$$\frac{\partial \tilde{c}^{ij}_{\mathbf{k}}}{\partial t} = \sum_{lm,\mathbf{k}'} A^{ij,lm}_{\mathbf{k},\mathbf{k}'}\tilde{c}^{lm}_{\mathbf{k}'} + K^{ij}_{\mathbf{k}}, \tag{65}$$

with lm and ij assuming the set of values 11, 22, 33, 12, 13, 23, 31, 21, 32, as

$$R^{ij,lm}_{\mathbf{k},\mathbf{k}'}(z) = \left(z\mathbf{I} - A^{ij,lm}_{\mathbf{k},\mathbf{k}'}\right)^{-1}, \tag{66}$$

I denoting the identity matrix.

With the self consistent solutions of equations (55) we then obtain the populations of the three bands in the presence of one or two driving fields and, using equation (64), we compute the absorption coefficient of the probe beam at the frequency ω_p.

We have performed the above calculations for the structure illustrated in Figure 1 considering an Asymmetric Double Quantum Well of n type GaAs/$Al_{0.3}Ga_{0.7}$As of thicknesses reported in Table 1, where all the material parameters are also given. We consider a low temperature condition ($T = 6$ K) and low impurity doping (electron density $n_e = 5\times10^{10}$ cm^{-2}), so that only the lowest band is populated with the Fermi energy given in Table 1.

We first consider the case of a single control beam of frequency $\Omega_2 = \omega_{32}$. The population of the bands is not modified by the control beam, because the two highest bands are not populated, but an eventual EIT must depend on the intensity of the control beam.

We give in Figure 2 the dephasing due to the electron-electron scattering in the three subbands. In this case the result is not modified by the control beam, since it does not change the populations. We observe that also in the higher subbands we obtain a dephasing due to intersubband interactions. As a consequence each subband contributes a broadening with an average value of about 0.5 ps^{-1} and a k dependence shown in the figure.

In Figure 3 we give the dephasing rates due to the electron-phonon scattering. We notice that for the states below the Fermi level the dephasing is such as to give a longer lifetime for the subband 2 with respect to that of subband 3.

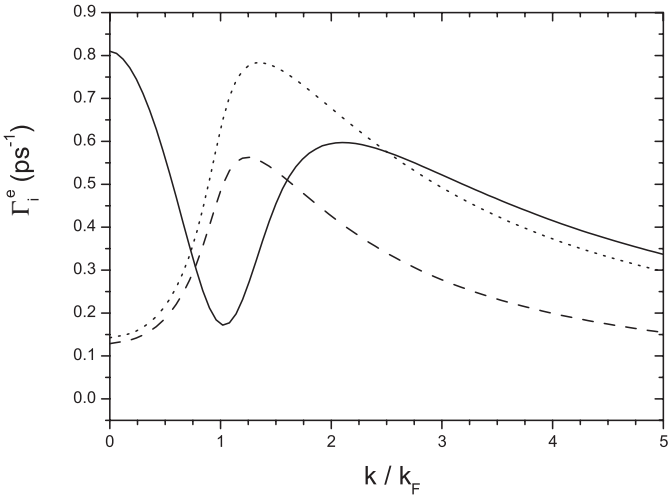

Fig. 2. Case of a single control beam ($\Omega_2 \approx \omega_{32}$, $\alpha = 0$). Dephasing $\Gamma_i^{\rm e}(k)$ due to the electron-electron scattering in the three subbands as a function of k. Solid line refers to the lowest subband. The dotted and the dashed lines refer to bands 3 and 2 respectively.

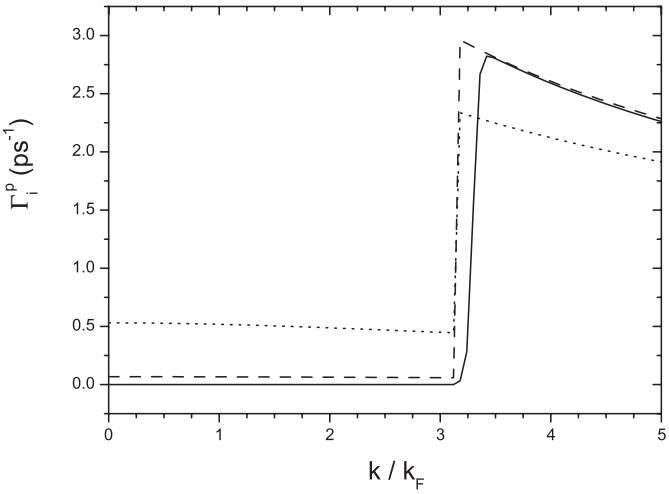

Fig. 3. Case of a single control beam ($\Omega_2 \approx \omega_{32}$, $\alpha = 0$). Electron-phonon scattering rates $\Gamma_i^{\rm p}(k)$. The dotted and the dashed lines refer to bands 3 and 2 respectively; the continuous line to band 1. A large increase in the broadening occurs when $\hbar^2k^2/2m_e^*$ is above $\hbar\omega_{\rm LO}$ for bands 2 and 3, and above $\hbar\omega_{\rm LO} + E_F$ for band 1, due to intrasubband electron-phonon scattering.

In Figure 4 we give the total dephasing rates due to the electron-electron and electron-phonon scattering. The curves are the sums of the ones given in Figures 2 and 3. We observe that the ratio Γ_3 / Γ_2 is about 3 for the states below the Fermi level which are the ones relevant for optical transitions. This is the border region to give a coherent transparency. The situation is less favorable than in atomic cases, where the lifetimes of level 2 is many orders of magnitude larger than that of level 3, because is mostly due to spontaneous emission. In the example of Sadeghi *et al.* [5] a ratio of about 1000 was assumed and a very neat EIT was obtained.

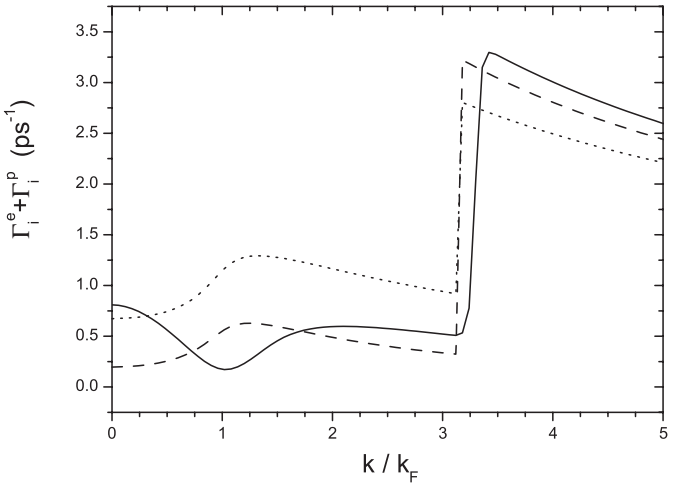

Fig. 4. Case of a single control beam ($\Omega_2 \approx \omega_{32}$, $\alpha = 0$). Total scattering rates $\Gamma_i(k) = \Gamma_i^{\rm p}(k) + \Gamma_i^{\rm e}(k)$. The dotted and the dashed lines refer to bands 3 and 2 respectively. The continuous line to band 1. The curves are the sums of those given in Figures 2 and 3. We notice that for small **k** vectors $\Gamma_3(k) > \Gamma_2(k)$, and also $\Gamma_1(k)$ is relevant due to intrasubband scattering.

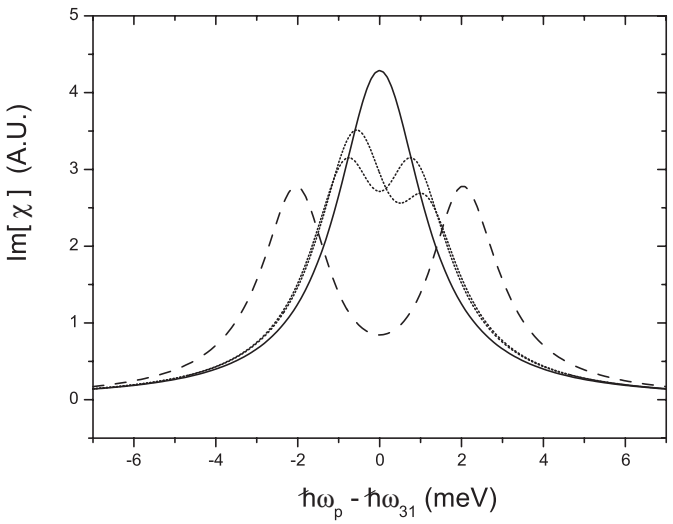

Fig. 5. Imaginary part of the susceptibility with a single resonant control beam ($\Omega_2 = \omega_{32}$, $\alpha = 0$) and different coupling field intensities: $\beta = 0$ (solid line), 0.75 (symmetric dotted line), and 2 (dashed line) ps^{-1}. The asymmetric dotted line is obtained for $\beta = 0.75$ ps^{-1} and a detuning of the coupling field $\Omega_2 - \omega_{32} = 0.5$ meV.

With the computed values appropriate to our QW we obtain the absorption spectra shown in Figure 5 for different intensities of the control beam. With a relatively weak control beam ($\beta \approx 0.75$ ps^{-1}, corresponding to about 0.1 MW cm^{-2}) the deep is totally inside the linewidth, so that the transparency is due to the interference in the transition probability amplitudes at different orders. We notice that the coherence of the EIT effects is demonstrated by the fact that the lineshape cannot be fitted with the sum of two absorption peaks, but the transparency is deeper. It is to be observed that a detuning of the control beam moves the absorption deep accordingly ($\omega_p - \Omega_2 = \omega_{21}$), as expected for coherent EIT. With a larger intensity of the control beam the transparency increases, but its width is larger than the absorption width. This corresponds to a dynamic Stark effect, which splits the levels of the uppermost band. We also notice that the sum rules for the nonlinear susceptibility must be obeyed [31].

We next consider the case of two control beams with frequencies $\Omega_1 = \omega_{31}$ and $\Omega_2 = \omega_{32}$, as suggested for atomic systems by Narducci *et al.* [17]. The main difference from the case of one control beam is that in this case the steady state population of the bands is strongly modified, and also the dephasing rates are modified. For the purpose of exemplification we present in Figure 6 the final population of the three subbands for the case $\alpha = \beta = 0.1$ ps^{-1}. We can observe that the final solution required by consistency between dephasing and population of the states, gives an appreciable redistribution of the electron population on the different subbands, even with a relatively small intensity of the control beams, the highest subband remaining essentially depopulated due to coherence effects. The effect of the population redistribution on the electron-phonon and electron-electron dephasing is

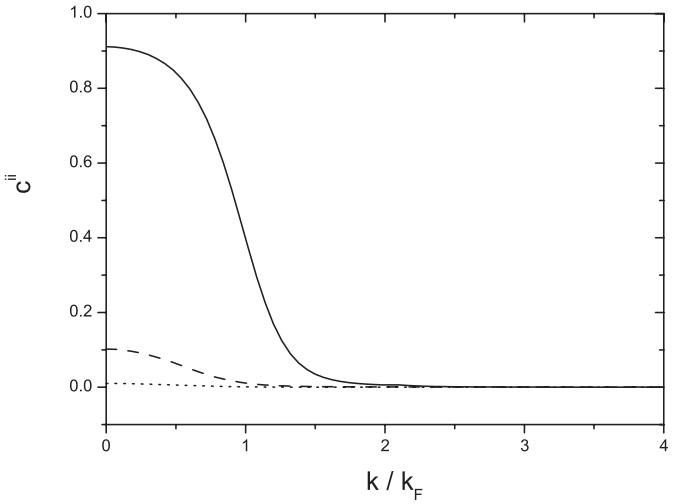

Fig. 6. Electron density distribution in the 1st (solid line), 2nd (dashed line) and 3rd (dotted line) subband for two resonant coupling fields ($\Omega_1 = \omega_{31}$, $\Omega_2 = \omega_{32}$) with $\alpha = 0.1$ ps^{-1} and $\beta = 0.1$ ps^{-1}.

shown in Figure 7, where we give the total dephasing rates. We observe that due to the increased electron-electron scattering rates, $\Gamma_3 \approx \Gamma_2$, so that the narrow transparency is not observable. However we obtain a large enhanced transparency, which is shown in Figure 8. We notice that the computed absorption of the probe beam shows a considerable decrease which follows from the population redistribution. The EIT has a quite different lineshape from that observed in the case of one driving field of frequency ω_{32}, the absorption being greatly reduced over the entire spectrum. This is due to the broadening of the probe transparency because of electron-electron scattering.

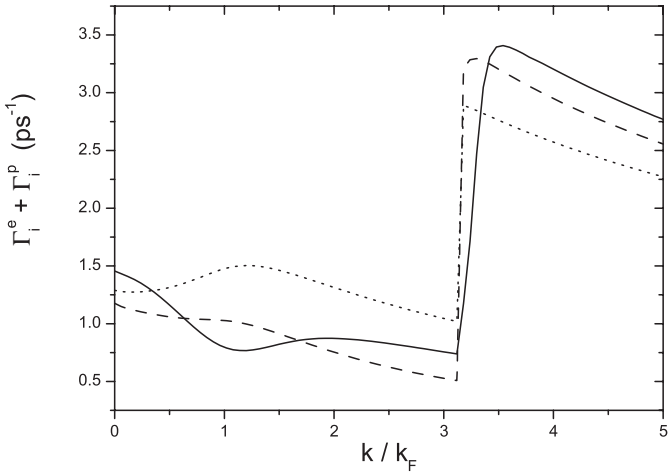

Fig. 7. Total scattering rates $\Gamma_i(k) = \Gamma_i^{\rm p}(k) + \Gamma_i^{\rm e}(k)$ for two resonant coupling fields with the intensities given in Figure 6. The dotted and the dashed lines refer to bands 3 and 2 respectively. The continuous line to band 1. Notice how at small k values $\Gamma_3(k) \approx \Gamma_2(k)$.

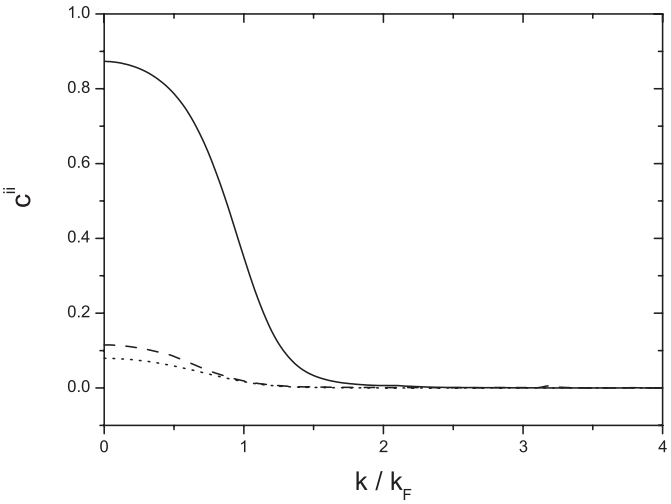

Fig. 9. Electron density distribution in the 1st (solid line), 2nd (dashed line) and 3rd (dotted line) subband for one resonant coupling field: $\Omega_1 = \omega_{31}$, $\beta = 0$ and $\alpha = 0.1$ ps^{-1}.

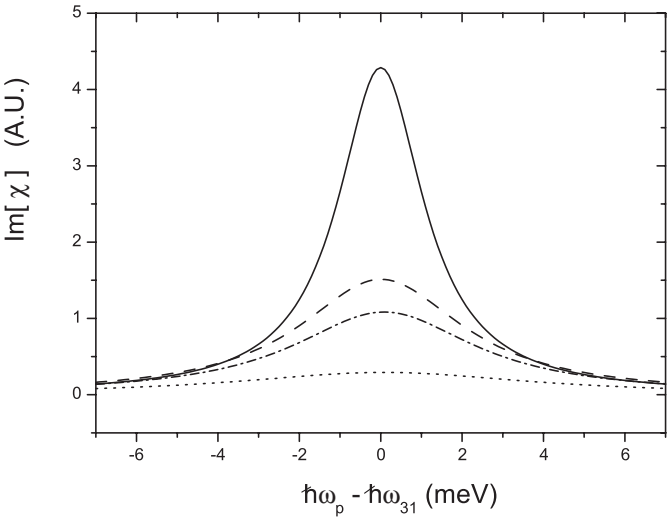

Fig. 8. Imaginary part of the susceptibility for different values of the resonant coupling field intensities ($\Omega_1 = \omega_{31}$, $\Omega_2 = \omega_{32}$): $\alpha = \beta = 0$ (solid line), 0.1 (dashed line) and 0.3 ps^{-1} (dotted line). The dash dotted line refers to the case of a single coupling field resonant between subband 1 and 3: $\Omega_1 = \omega_{31}$, $\beta = 0$ and $\alpha = 0.1$ ps^{-1}.

We can observe that the quantity $\int \omega {\rm Im}\chi(\omega){\rm d}\omega$ is not conserved in this case because we have limited our calculation to a small portion of the spectrum. When the population is changed, transitions to other subbands should be considered from all our subbands to satisfy the sum rules.

The effects of the coherence between the control beams can be seen by comparing the absorption obtained with a driving field of frequency ω_{31} ($\alpha = 0.1$ ps^{-1}, dash-dotted line of Fig. 6) and the absorption obtained with both resonant driving fields ($\alpha = \beta = 0.1$ ps^{-1}, dashed line of Fig. 8). When only a single control beam at frequency $\Omega_1 = \omega_{31}$ is applied we obtain an even larger decrease in absorption due to saturation because the third subband is partly populated. We also point out that, due to scattering with phonons, a great part of the electrons end up in the intermediate subband, as shown in Figure 9. When we apply two control beams, they interfere and are very effective in driving the electrons to the intermediate subband, so that the highest one remains practically empty, as illustrated in Figure 6. This coherence effect produces a smaller transparency than that obtained with one control beam of frequency ω_{31}, because in the latter case the third subband is partly populated.

6 Conclusions and discussion

We have shown that properly chosen double asymmetric quantum wells can sustain the phenomenon of EIT, when driving fields resonant with intersubband transitions are applied.

We have considered a configuration with the intermediate subband closer in energy to the highest subband than to the lowest one. The dephasing rates due to electron-electron and electron-phonon interaction are computed, taking into account the occupation of the band states. A system of coupled equations for electron density matrices are derived, including all the dephasing rates. From them the absorption of a probe beam nearly resonant between the lowest and the highest subband can be computed, for any given substance and well widths, as a function of the intensity of the driving fields.

Numerical calculations on GaAs/$Al_{0.3}Ga_{0.7}$As QW are carried out with one and with two driving fields. In the former case the population of the subbands is practically unchanged and we find a well defined EIT for appropriate intensities of the control beam.

In the case of two driving fields some electrons occupy the higher subbands producing a kind of population trapping. This redistribution of the electrons has two effects that combine to give a greatly enhanced transparency. First, the presence of electrons in the higher subbands reduces the total absorption in the $\hbar\omega_{31}$ region. In addition the dephasing rates are greatly increased because of the electron-electron scattering in all the bands, and this modifies the lineshape of the absorption: the transparency hole is modified into a much broader more intense transparency. As a consequence the EIT observed is qualitatively different from the narrow usual EIT.

The present results confirm the possibility of obtaining EIT in solids, using asymmetric double quantum wells. Though the transparency is not as complete as in high density atomic systems, the advantage of obtaining EIT and the consequent hyperanomalous dispersion in solid materials can be of great relevance in related phenomena (orders of magnitude decrease in the group velocity, Fizeau drag anomalies, Cerenkov cone modification, superluminar propagation). The enhanced transparency with two driving fields and the associate emptiness of the highest subband can also be of interest in practical applications.

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Appendix A: Absorption spectra

We consider a linear response of the system at a probe field (57) at frequency ω_p, near the transition frequency ω_{31}. In the following we compute the rate of absorption of energy from the perturbing field. The absorption is proportional to the imaginary part of the linear susceptibility. To determine the rate of absorption, we must find the change produced by an perturbation in the equilibrium density operator. In the lowest order this change may be expressed in terms of the interaction Hamiltonian $H'(t)$ associated with the perturbation and the unperturbed equilibrium density operator ρ as

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho = \frac{\mathrm{i}}{\hbar}[H,\rho],$$
$$\Delta\,\rho(t) = \frac{\mathrm{i}}{\hbar}\int_{-\infty}^{t}\mathrm{d}t'\,[H'(t'),\rho]. \tag{A.1}$$

The operator ρ in the above equation is considered as a time independent quantity. The rate at which the perturbation does work on the system is proportional to the imaginary part of the susceptibility [28], and is of the form [29]

$$\begin{aligned} W'(t) &= \mathrm{Tr}\left[\Delta\,\rho(t)\,\frac{\partial H'(t)}{\partial t}\right] \\ &= \mathrm{Tr}\left[\left(\frac{\mathrm{i}}{\hbar}\int_{-\infty}^{t}\mathrm{d}t'\,[H'(t'),\rho]\right)\,\frac{\partial H'(t)}{\partial t}\right] \\ &= \frac{1}{\mathrm{i}\hbar}\int_{-\infty}^{t}\mathrm{d}t'\,\mathrm{Tr}\left\{\rho\left[\frac{\partial H'(t)}{\partial t}\,,H'(t')\right]\right\}, \end{aligned} \tag{A.2}$$

where we used the property of the invariance of the trace operation under a cyclic permutation.

We consider the perturbing Hamiltonian (59) and perform the time derivative to obtain

$$\frac{\partial H'}{\partial t} = \sum_{\mathbf{k}}\frac{\mathrm{i}\omega_p}{2}\left[M_0^{31}\mathcal{E}_0\,\mathrm{e}^{-\mathrm{i}\omega_p t}\hat{c}_{\mathbf{k}}^{31} - M_0^{13}\mathcal{E}_0^*\,\mathrm{e}^{\mathrm{i}\omega_p t}\hat{c}_{\mathbf{k}}^{13}\right], \tag{A.3}$$

with

$$\hat{c}_{\mathbf{k}}^{13} = \hat{c}_{\mathbf{k}}^{1\dagger}\,\hat{c}_{\mathbf{k}}^{3}, \quad \hat{c}_{\mathbf{k}}^{31} = \hat{c}_{\mathbf{k}}^{3\dagger}\,\hat{c}_{\mathbf{k}}^{1}. \tag{A.4}$$

Next we compute the commutator

$$\begin{aligned} &\left[\frac{\partial H'(t)}{\partial t}\,,H'(t')\right] = \\ &\sum_{\mathbf{k},\mathbf{k}'}\Bigg\{-\frac{\mathrm{i}\omega_p}{2}\left[M_0^{31}\mathcal{E}_0\,\mathrm{e}^{-\mathrm{i}\omega_p t}\hat{c}_{\mathbf{k}}^{31}(t) - M_0^{13}\mathcal{E}_0^*\,\mathrm{e}^{\mathrm{i}\omega_p t}\hat{c}_{\mathbf{k}}^{13}(t)\right] \\ &\times\left[M_0^{31}\mathcal{E}_0\,\mathrm{e}^{-\mathrm{i}\omega_p t'}\hat{c}_{\mathbf{k}'}^{31}(t') + M_0^{13}\mathcal{E}_0^*\,\mathrm{e}^{\mathrm{i}\omega_p t'}\hat{c}_{\mathbf{k}'}^{13}(t')\right] \\ &+\mathrm{i}\omega_p\left[M_0^{31}\mathcal{E}_0\,\mathrm{e}^{-\mathrm{i}\omega_p t'}\hat{c}_{\mathbf{k}'}^{31}(t') + M_0^{13}\mathcal{E}_0^*\,\mathrm{e}^{\mathrm{i}\omega_p t'}\hat{c}_{\mathbf{k}'}^{13}(t')\right] \\ &\times\left[M_0^{31}\mathcal{E}_0\,\mathrm{e}^{-\mathrm{i}\omega_p t}\hat{c}_{\mathbf{k}}^{31}(t) - M_0^{13}\mathcal{E}_0^*\,\mathrm{e}^{\mathrm{i}\omega_p t}\hat{c}_{\mathbf{k}}^{13}(t)\right]\Bigg\} \\ &= \sum_{\mathbf{k}}\mathrm{i}\omega_p\frac{\left|\mathcal{E}_0 M_0^{31}\right|^2}{2}\Big[-\mathrm{e}^{\mathrm{i}\omega_p(t'-t)}\hat{c}_{\mathbf{k}}^{31}(t)\hat{c}_{\mathbf{k}}^{13}(t') \\ &+\mathrm{e}^{\mathrm{i}\omega_p(t-t')}\hat{c}_{\mathbf{k}}^{13}(t)\,\hat{c}_{\mathbf{k}}^{31}(t') - \mathrm{e}^{\mathrm{i}\omega_p(t-t')}\hat{c}_{\mathbf{k}}^{31}(t')\hat{c}_{\mathbf{k}}^{13}(t) \\ &+\mathrm{e}^{\mathrm{i}\omega_p(t'-t)}\hat{c}_{\mathbf{k}}^{13}(t')\hat{c}_{\mathbf{k}}^{31}(t)\Big], \end{aligned} \tag{A.5}$$

and insert this expression into (A.2), to obtain

$$\begin{aligned} W' = \sum_{\mathbf{k}}\frac{\omega_p}{\hbar}\left|\mathcal{E}_0 M_0^{31}\right|^2\int_{-\infty}^{t}\mathrm{d}t'\Big[&-\mathrm{e}^{\mathrm{i}}\omega_p(t'-t)\left\langle\hat{c}_{\mathbf{k}}^{31}(t)\hat{c}_{\mathbf{k}}^{13}(t')\right\rangle \\ &+\mathrm{e}^{\mathrm{i}\omega_p(t-t')}\left\langle\hat{c}_{\mathbf{k}}^{13}(t)\,\hat{c}_{\mathbf{k}}^{31}(t')\right\rangle - \mathrm{e}^{\mathrm{i}\omega_p(t-t')}\left\langle\hat{c}_{\mathbf{k}}^{31}(t')\hat{c}_{\mathbf{k}}^{13}(t)\right\rangle \\ &+\mathrm{e}^{\mathrm{i}\omega_p(t'-t)}\left\langle\hat{c}_{\mathbf{k}}^{13}(t')\hat{c}_{\mathbf{k}}^{31}(t)\right\rangle\Big]. \end{aligned} \tag{A.6}$$

Performing the integrations we obtain

$$
\begin{aligned}
-\int_{-\infty}^{t} \mathrm{d}t' \mathrm{e}^{\mathrm{i}\omega_p(t'-t)} \left\langle \hat{c}_{\mathbf{k}}^{31}(t)\, \hat{c}_{\mathbf{k}}^{13}(t') \right\rangle &= -\int_{0}^{\infty} \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\omega_p\tau} g_{\mathrm{d}}(\tau), \\
\int_{-\infty}^{t} \mathrm{d}t'\, \mathrm{e}^{\mathrm{i}\omega_p(t-t')} \left\langle \hat{c}_{\mathbf{k}}^{13}(t)\, \hat{c}_{\mathbf{k}}^{31}(t') \right\rangle &= \int_{-\infty}^{0} \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\omega_p\tau}\, g_e(\tau), \\
-\int_{\infty}^{t} \mathrm{d}t' \mathrm{e}^{\mathrm{i}\omega_p(t-t')} \left\langle \hat{c}_{\mathbf{k}}^{31}(t') \hat{c}_{\mathbf{k}}^{13}(t) \right\rangle &= -\int_{-\infty}^{0} \mathrm{d}\tau \mathrm{e}^{-\mathrm{i}\omega_p\tau} g_d(\tau), \\
\int_{-\infty}^{t} \mathrm{d}t' \mathrm{e}^{\mathrm{i}\omega_p(t'-t)} \left\langle \hat{c}_{\mathbf{k}}^{13}(t') \hat{c}_{\mathbf{k}}^{31}(t) \right\rangle &= \int_{0}^{\infty} \mathrm{d}\tau \mathrm{e}^{-\mathrm{i}\omega_p\tau} g_e(\tau),
\end{aligned}
\tag{A.7}
$$

where we used the correlation functions

$$
\begin{aligned}
g_e(\tau) &= \left\langle \hat{c}_{\mathbf{k}}^{13}(t') \hat{c}_{\mathbf{k}}^{31}(\tau + t') \right\rangle, \\
g_d(\tau) &= \left\langle \hat{c}_{\mathbf{k}}^{31}(\tau + t') \hat{c}_{\mathbf{k}}^{13}(t') \right\rangle \cdot
\end{aligned}
\tag{A.8}
$$

Putting all the above integrals in the expression (A.6) we obtain for this model system the absorption to be proportional to the following Fourier transform:

$$
W' = \sum_{\mathbf{k}} \omega_p \frac{\left|\mathcal{E}_0 M_0^{13}\right|^2}{2\hbar} \int_{-\infty}^{\infty} \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\omega_p\tau} \left(g_e(\tau) - g_d(\tau)\right). \tag{A.9}
$$

Including the factor $|M_0^{13}|^2$ into the correlation functions and making use of the expressions (60) we arrive at

$$
W' = \omega_p \frac{|\mathcal{E}_0|^2}{2\hbar} \sum_{\mathbf{k}} \int_{-\infty}^{\infty} \mathrm{d}\tau\, \mathrm{e}^{-\mathrm{i}\omega_p\tau} \left\langle \left[\hat{p}_{\mathbf{k}}^{-}(t'), \hat{p}_{\mathbf{k}}^{+}(\tau + t')\right] \right\rangle, \tag{A.10}
$$

which can be written in terms of the Laplace transform as

$$
W' = \frac{\omega_p}{\hbar} |\mathcal{E}_0|^2 \sum_{\mathbf{k}} \mathrm{Re}\left\{\mathcal{L}[G(\tau); s = \mathrm{i}\omega_p]\right\}, \tag{A.11}
$$

with (63)

$$
G(\tau) = \sum_{\mathbf{k}} G_{\mathbf{k}}(\tau, t') = \sum_{\mathbf{k}} \left\langle \left[\hat{p}_{\mathbf{k}}^{-}(t'), \hat{p}_{\mathbf{k}}^{+}(\tau + t')\right] \right\rangle \cdot \tag{A.12}
$$

The imaginary part of the susceptibility is related to the dissipated energy and can be written in the form (61). To evaluate the commutator we make use of the quantum regression theorem [5,30], which states that if we know the mean value of an operator $S(t)$ as a linear combination of the mean values of a set of operators $S_\mu(t')$ with $t' < t$,

$$
\langle S(t) \rangle = \sum_{\mu} \left[\mathcal{O}_\mu(t, t') \langle S_\mu(t') \rangle + \lambda_\mu\right], \tag{A.13}
$$

where $t > t'$, then we can calculate the mean value of the product $S(t)N(t')$ as

$$
\langle S(t)N(t') \rangle = \sum_{\mu} \left[\mathcal{O}_\mu(t, t') \langle S_\mu(t')N(t') \rangle + \lambda_\mu \langle N(t') \rangle\right] \cdot \tag{A.14}
$$

Performing a Laplace transform with respect to t, we obtain for expressions (A.13) and (A.14)

$$
\langle \mathcal{L}[S(t); z] \rangle = \sum_{\mu} \left[\mathcal{L}[\mathcal{O}_\mu(t, t'); z] \langle S_\mu(t') \rangle + \lambda_\mu\right], \tag{A.15}
$$

$$
\begin{aligned}
\langle \mathcal{L}[S(t); z] N(t') \rangle = \sum_{\mu} \big[&\mathcal{L}[\mathcal{O}_\mu(t, t'); z] \langle S_\mu(t')N(t') \rangle \\
&+ \lambda_\mu \langle N(t') \rangle\big].
\end{aligned}
\tag{A.16}
$$

In order to use this theorem to evaluate the Laplace transforms of $\langle \hat{p}_k^-(t') \hat{p}_k^+(\tau + t') \rangle$ and $\langle \hat{p}_k^+(\tau + t') \hat{p}_k^-(t') \rangle$ we first need to identify the operator $S(t)$ and a set of basis operators mean values $\{\langle S_\mu \rangle\}$. We choose $S(t) = \hat{p}_{\mathbf{k}}^+(t)$ and the mean values $\tilde{c}_{\mathbf{k}}^{ij}$ which can be obtained from equation (55). Then we identify $\mathcal{O}_\mu$, which obtains directly from the solutions of equations (55), expressed in the form (65). This equations can be solved by means of the Laplace transform, yielding in the steady state limit ($t \to \infty$)

$$
\begin{aligned}
\langle \mathcal{L}[\hat{p}_k^+(t); s = \mathrm{i}\omega_p] \rangle &= M_0^{31} \mathcal{L}[c_k^{31}; s = \mathrm{i}\omega_p] \\
&= M_0^{31} \mathcal{L}[\tilde{c}_k^{31}; s = \mathrm{i}\omega_p - i\Omega_1] \\
&= M_0^{31} \sum_{k',\mu} \left(R_{k,k'}^{31,\mu}(z) \tilde{c}_{k'}^{\mu}(t') + \frac{1}{z} R_{k,k'}^{31,\mu}(z) K_{k'}^{\mu} \right),
\end{aligned}
\tag{A.17}
$$

where the tensor $R_{k,k'}^{ij,lm}(z)$ is defined by equation (66) and $z = \mathrm{i}\omega_p - \mathrm{i}\Omega_1$. We can now use the above theorem (A.15–A.16) and put

$$
\begin{aligned}
&\hat{p}_k^-(t') \equiv N(t'), \\
&\mathcal{L}[\mathcal{O}_{\mu,k'}(t, t'); z] = M_0^{31} R_{k,k'}^{31,\mu}(z), \\
&\lambda_{\mu,k'} = \frac{M_0^{31}}{z} R_{k,k'}^{31,\mu}(z) K_{k'}^{\mu},
\end{aligned}
\tag{A.18}
$$

to obtain, in the steady state limit $t, t' \to \infty$,

$$
\begin{aligned}
\mathcal{L}\left[\left\langle \hat{p}_{\mathbf{k}}^+(\tau + t') \hat{p}_{\mathbf{k}}^-(t') \right\rangle; s = \mathrm{i}\omega_p\right] = \left|M_0^{31}\right|^2 \Bigg(&R_{k,k}^{31,31}(z) \tilde{c}_k^{33} \\
&+ R_{k,k}^{31,11}(z) \tilde{c}_k^{13} + R_{k,k}^{31,21}(z) \tilde{c}_k^{23} + \sum_{k',\mu} \lambda_{\mu,k'} \tilde{c}_k^{13} \Bigg).
\end{aligned}
\tag{A.19}
$$

Using the same procedure to evaluate $\mathcal{L}[\langle \hat{p}_k^-(t') \hat{p}_k^+(\tau + t') \rangle; s = \mathrm{i}\omega_p]$, we obtain

$$
\begin{aligned}
\mathcal{L}[\left\langle \hat{p}_k^-(t') \hat{p}_k^+(\tau + t') - \hat{p}_k^+(\tau + t') \hat{p}_k^-(t') \right\rangle; s = \mathrm{i}\omega_p] = \\
- \left|M_0^{13}\right|^2 \Big[&R_{k,k}^{31,31}(z) \left(\tilde{c}_k^{33} - \tilde{c}_k^{11}\right) \\
&+ \left(R_{k,k}^{31,11}(z) - R_{k,k}^{31,33}(z)\right) \tilde{c}_k^{13} \\
&+ R_{k,k}^{31,21}(z) \tilde{c}_k^{23} - R_{k,k}^{31,32}(z) \tilde{c}_k^{12} \Big],
\end{aligned}
\tag{A.20}
$$

with $z = \mathrm{i}\omega_p - \mathrm{i}\Omega_1$, which can be inserted in equation (61) to obtain the final expression (64).

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